

## “More Problems Than You Can Shake a Stick At” (Studying for the Projectile Motion Test)

A note before you start...

Most of the questions are very basic and straight forward vector and projectile motion questions. That's all there is to it. You have to know the basics and know them perfectly.

These pages are designed to give you enough practice that you can master the basics. The keys are on the page following each set of problems.

Included:

Page 2—Vector Basics—Breaking up vectors using sin and cos. How to put x and y components back together to find the resultant's magnitude and direction. Don't skip this page. Most of you make mistakes doing this process.

Page 3—key

Page 4—Adding Vectors. I used my Excel calculator program to crunch all these numbers for you. There is no reason why you should miss the adding vector problem, but many of you do anyway. There are enough problems that if you can do all of these you will be perfect (and bored).

Page 5—key

Page 6—Relative Motion Example 1 Work in x and y independently.

Page 7—key

Page 8—Relative Motion Ex 2 and 3.

Page 9—Key

Page 10—Projectile Motion Concepts. These were supposed to be easy points on the test, but ended up killing many of you.

Page 11—key

Page 12—Projectile Motion Problems—Again, there are enough to choke you with. Work 'em until you are a “Projectile Master”.

Page 13—key

## Vector Basics

1. I walk 35 m at  $32^\circ$ . How far east and north do I go?
2. A plane flies 980 miles at  $76^\circ$ . How far east and north did it fly?
3. A porcupine waddles 13 meters at a direction of  $16^\circ$ . How far east and north did it walk?
4. A boat moves 812 km at  $230^\circ$ . Give the how far the boat moved in the x and y directions.
5. A slingshot throws a ball 72 yards at an angle of  $310^\circ$ . Give the x and y components of its displacement.
6. A salamander scampers 22.6 m to avoid a rattle snake. If the slimy amphibian moved at a direction of  $125^\circ$ , how far did it move in the x and y directions from its initial position?
7. If an object moves the following amounts, find the object's total displacement (magnitude and direction). Double check your direction by figuring out what quadrant it is in.
  - A. 8 m east and 4 m north.
  - B. 4 m west and 9 m south.
  - C. 6 m west and 8 m north
  - D. 7 m south and 4 m east.

Answers:

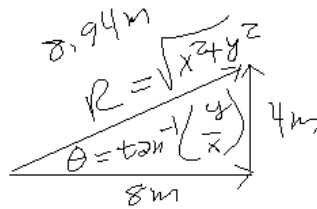
**Vector Problems:**

1. I walk 35 m at  $32^\circ$ . How far east and north do I go?  
 $X = 29.86\text{m}$        $Y = 18.55\text{m}$
2. A plane flies 980 mph at  $76^\circ$ . How fast did it fly east and north?  
 $V_x = 237.08\text{ mph}$        $V_y = 950.89\text{ mph}$
3. A porcupine waddles 13 meters at a direction of  $16^\circ$ . How far east and north did it walk?  
 $X = 12.50\text{ m}$        $Y = 3.58\text{ m}$
4. A boat moves 812 km at  $230^\circ$ . Give the how far the boat moved in the x and y directions.  
 $X = -521.94\text{km}$        $Y = -622.03\text{ km}$
5. A slingshot throws a ball 72 yards at an angle of  $310^\circ$ . Give the x and y components of its displacement.  
 $X = 46.28\text{ yards}$        $Y = -55.16\text{ yards}$
6. A salamander scampers 22.6 cm/sec to avoid a rattle snake. If the slimy amphibian moved at a direction of  $125^\circ$ , how fast did it move in the x and y directions from its initial position?  
 $V_x = -12.96\text{ cm/sec}$        $V_y = 18.51\text{ cm/sec}$

7A 8 m east 4 m north:

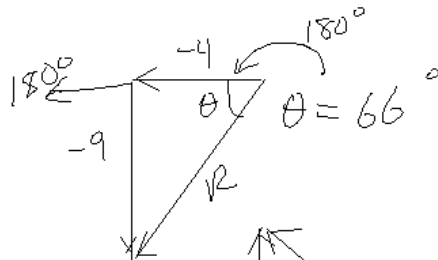
①

$R = 8.94$  at  $26.6^\circ$



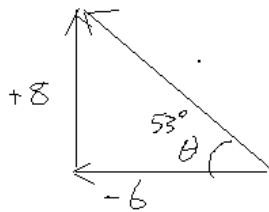
7B 4 m west 9 m south:

$R = 9.85\text{m}$  at  $246^\circ$



7C 6 m west 8 m north:

$R = 10\text{ m}$  at  $127^\circ$



7D 7 m south and 4 m <sup>east</sup> ~~west~~:  $R = 8.06\text{m}$  at  $-60.3^\circ$

*Make sure you can do the vector basic problems before you try these.*

**Vector Addition:** Add the following pairs of vectors. All answers must have magnitude and direction.

1. A person walks 45 m at  $65^\circ$  and then turns to  $325^\circ$  and walks 122 m. Find the person's total displacement (magnitude and direction).
2. 340m at  $12^\circ$ , then 733m at  $335^\circ$ . Find total displacement.
3. 182 m at  $34^\circ$ , then 89m at  $97^\circ$ .
4. 1.47 feet at  $7^\circ$ , then 2.38 feet at  $342^\circ$ .
5. 366 yards at  $83^\circ$ , then 472 yards at  $28^\circ$ .
6. 135 miles at  $154^\circ$ , then 87 miles at  $286^\circ$ .
7. 56 inches at  $17^\circ$ , then 27 inches at  $189^\circ$ .

Answers:

**Vector Addition:** Add the following pairs of vectors. All answers must have magnitude and direction.

- 1 A person walks 45 m at 65 degrees then turns to 325 degrees and walks 122 meters. Find the total displacement of the person.

$$\begin{aligned}
 &45 \text{ at } 65^\circ & X_1 &= 45 \cos 65^\circ = 19.02 \text{ m} & Y_1 &= 45 \sin 65^\circ = 40.78 \text{ m} \\
 &122 \text{ m at } 325^\circ & X_2 &= \cos = 99.94 \text{ m} & Y_2 &= \sin = -69.98 \text{ m} \\
 && X_t &= 118.95 \text{ m} & Y_t &= -29.19 \text{ m} \\
 && R &= 122.48 \text{ m} \\
 && & \text{at } -13.79^\circ
 \end{aligned}$$

2. 340m at 12°, then 733m at 335°.

|        | X      | Y       |         |           |
|--------|--------|---------|---------|-----------|
| V1     | 332.57 | 70.69   |         |           |
| V2     | 664.32 | -309.78 | Mag     | Direction |
| Totals | 996.87 | -239.09 | 1025.2m | -13.49°   |

3. 182 m at 34°, then 89m at 97°.

|        | X      | Y      |         |           |
|--------|--------|--------|---------|-----------|
| V1     | 150.88 | 101.77 |         |           |
| V2     | -10.85 | 88.34  | Mag     | Direction |
| Totals | 140.04 | 190.11 | 236.12m | 53.62°    |

4. 1.47 feet at 7°, then 2.38 feet at 342°.

|        | X     | Y      |        |           |
|--------|-------|--------|--------|-----------|
| V1     | 1.459 | .179   |        |           |
| V2     | 2.26  | -.735  | Mag    | Direction |
| Totals | 3.722 | -0.556 | 3.76ft | -8.5°     |

5. 366 yards at 83°, then 472 yards at 28°.

|        | X      | Y      |        |           |
|--------|--------|--------|--------|-----------|
| V1     | 44.60  | 363.27 |        |           |
| V2     | 416.75 | 221.59 | Mag    | Direction |
| Totals | 461.36 | 584.86 | 744.92 | 51.7°     |

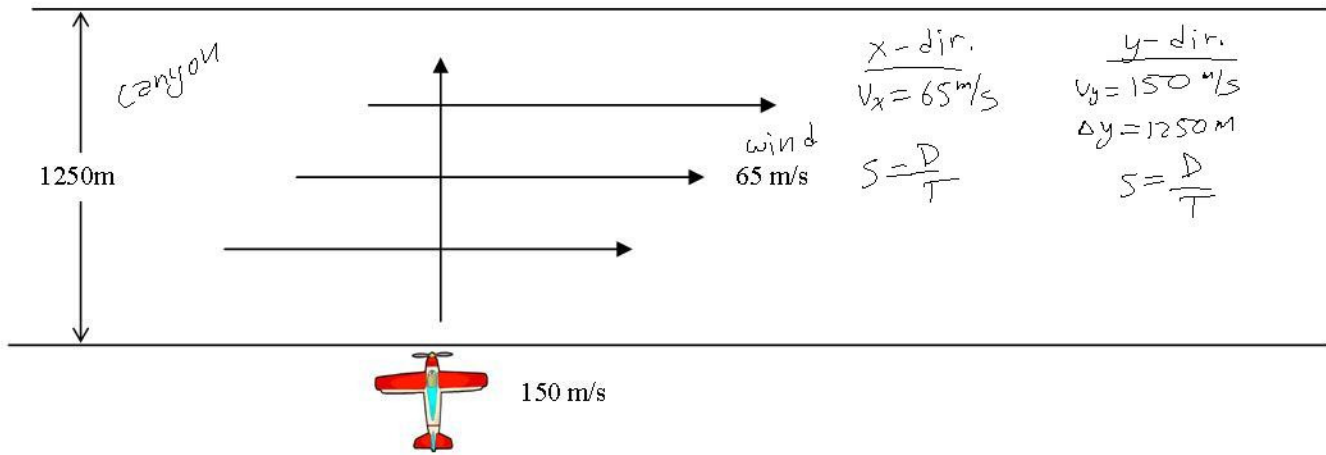
6. 135 miles at 154°, then 87 miles at 286°.

|        | X       | Y      |        |           |
|--------|---------|--------|--------|-----------|
| V1     | -121.34 | 59.18  |        |           |
| V2     | 23.98   | -83.63 | Mag    | Direction |
| Totals | -97.36  | -24.45 | 100.38 | 194°      |

7. 56 inches at 17°, then 27 inches at 189°.

|        | X      | Y     |       |           |
|--------|--------|-------|-------|-----------|
| V1     | 53.55  | 16.37 |       |           |
| V2     | -26.67 | -4.22 | Mag   | Direction |
| Totals | 26.89  | 12.15 | 29.50 | 24.3°     |





A plane has a speed of 150 m/s and is flying directly north ( $90^\circ$ ). It comes to a canyon that is orientated exactly east/west and has a width of 1250 m. The wind in the canyon is blowing directly east ( $0^\circ$ ) and blowing 65 m/s.

A. How long does it take for the plane to cross the canyon?

y-dir only

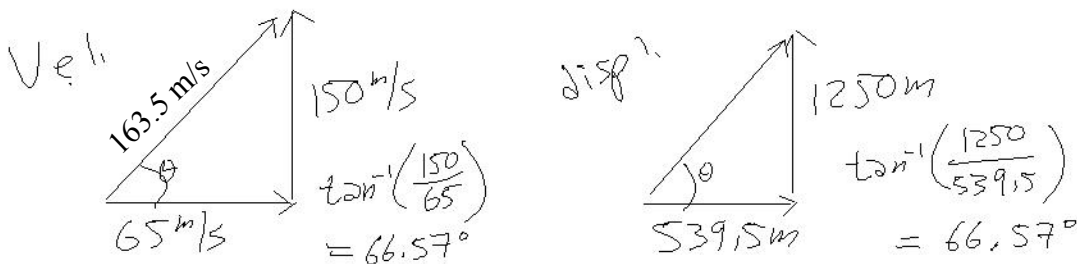
$$T = \frac{D}{S} = \frac{1250}{150} = 8.3 \text{ sec}$$

B. How far down the canyon is the plane pushed?

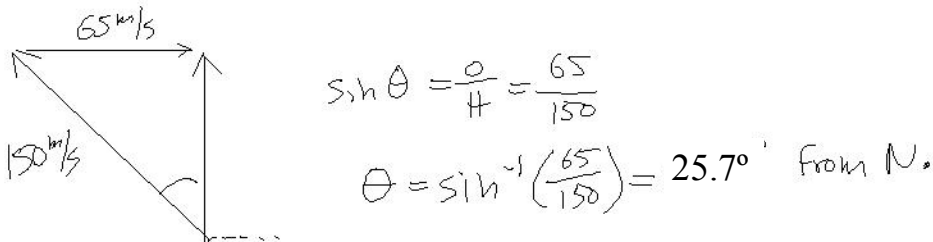
x-dir.

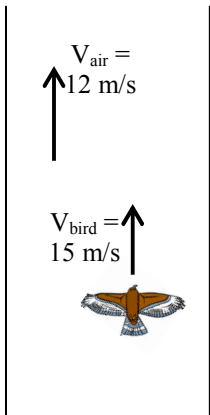
$$D = V_x t = 65(8.3) = 539.5 \text{ m}$$

C. What direction and speed does he actually fly?

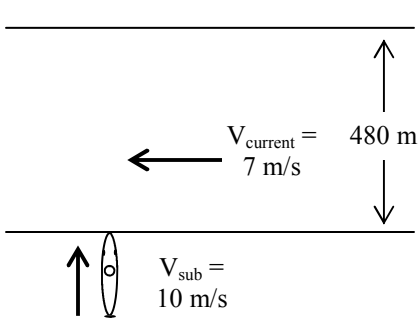
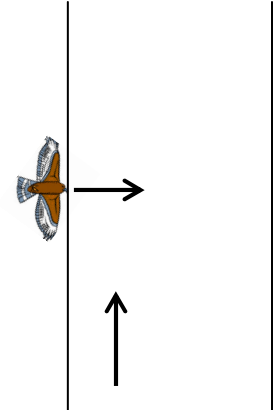


D. Which direction does he need to fly to end up directly across the canyon?





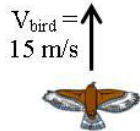
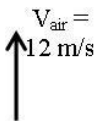
1. A bird has a velocity of 15 m/s in still air. The bird enters a canyon that has an airstream with a velocity of 12 m/s north.
  - A. What is the velocity of the bird relative to the ground if the bird flies with the air?
  - B. What is the velocity of the bird relative to the ground if the bird flies against the air?
  - C. What if the bird enters the air stream moving directly east?  
(Magnitude and direction, of course.)
  - D. If the canyon is 48 m wide, how long does it take the bird get across?
  - E. How far up the canyon has the bird been pushed by the air?
  - F. At what direction must the bird have to aim to get directly across the river.  
(Directly across the canyon is Lazy's path.)



A submarine on patrol comes across an underwater canyon that has a consistent current flowing thru it to the west.

- A. What is the velocity of the sub relative to the ground, if it turns and moves with the current (west)?
- B. What is the velocity of the sub relative to the ground, if it turns and moves against the current (east)?
- C. How long would it take to go 100 m west and then return?
- D. If the sub enters the air stream directly perpendicular to the current, what is its velocity and direction relative to the ground?
- E. If the canyon is 480 m wide, how long does it take the sub get across? (Hint: Is this an x or y-direction question? Then use only the information for that direction to solve.)
- F. How far along the canyon (west) has the sub drifted by the time it has crossed? (Again: x or y question?)
- G. At what direction must the sub have to aim to get directly across the canyon. (Directly across the canyon is Lazy's path.)





1. A bird has a velocity of 15 m/s in still air. The bird enters a canyon that has an airstream with a velocity of 12 m/s north.

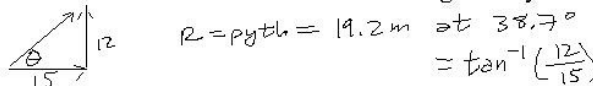
A. What is the velocity of the bird relative to the ground if the bird flies with the air?

$$15 + 12 = 27 \text{ m/s north (up) or } 90^\circ$$

B. What is the velocity of the bird relative to the ground if the bird flies against the air?

$$12 - 15 = -3 \text{ m/s or } 3 \text{ m/s south or } -90^\circ$$

C. What if the bird enters the air stream moving directly east? (Magnitude and direction, of course.)



D. If the canyon is 48 m wide, how long does it take the bird get across?

This is only an x-dir question, so use  $V_x$  and width (x)

$$S = \frac{D}{T} \quad T = \frac{D}{V_x} = \frac{48}{15} = 3.2 \text{ sec}$$

E. How far up the canyon has the bird been pushed by the air?

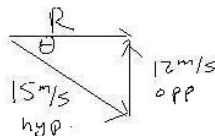
This is a y-dir. question, so use  $V_y$  and t to find y.

$$D = ST = V_y t = 12(3.2) = 38.4 \text{ m}$$

F. At what direction must the bird have to aim to get directly across the river.

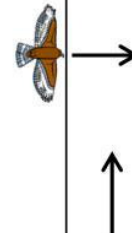
(Directly across the canyon is Lazy's path.)

Obviously the bird will have to turn into the current.

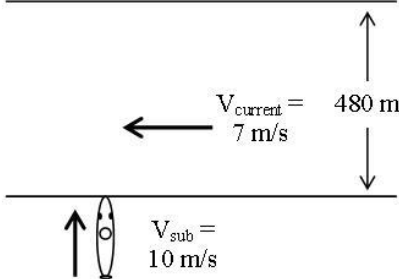


$$\sin \theta = \frac{12}{15}$$

$$\theta = \sin^{-1}\left(\frac{12}{15}\right) = -53.1^\circ \text{ OR } 53.1^\circ \text{ below the X axis}$$



A submarine on patrol comes across an underwater canyon that has a consistent current flowing thru it to the west.



A. What is the velocity of the sub relative to the ground, if it turns and moves with the current (west)?

$$17 \text{ m/s at } 180^\circ \text{ (west)}$$

B. What is the velocity of the sub relative to the ground, if it turns and moves against the current (east)?

$$3 \text{ m/s at } 0^\circ \text{ (east)}$$

C. How long would it take to go 100 m west and then return?

west is with current so  $S = \frac{D}{T}$

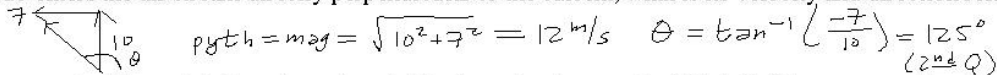
$$T = \frac{D}{S} = \frac{100}{17} = 5.9 \text{ sec}$$

the return is east  $V_{\text{east}} = 3 \text{ m/s}$

$$T = \frac{100}{3} = 33.3 \text{ sec}$$

total =  $5.9 + 33.3 = 39.2 \text{ sec}$

D. If the sub enters the air stream directly perpendicular to the current, what is its velocity and direction relative to the ground?



E. If the canyon is 480 m wide, how long does it take the sub get across? (Hint: Is this an x or y-direction question? Then use only the information for that direction to solve.)

y-dir question  $\Delta y = 480 \text{ m}$   $S = \frac{D}{T}$   $T = \frac{D}{S} = \frac{\Delta y}{V_y} = \frac{480}{10} = 48 \text{ sec}$

$V_y = 10 \text{ m/s (sub)}$

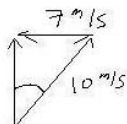
F. How far along the canyon (west) has the sub drifted by the time it has crossed? (Again: x or y question?)

x question  $t = 48 \text{ sec}$

$$V_x = 7 \text{ m/s} \quad S = \frac{D}{T} \quad D = ST = V_x t = 7(48) = 336 \text{ m}$$

G. At what direction must the sub have to aim to get directly across the canyon. (Directly across the canyon is Lazy's path.)

Sub will have to turn into the current.

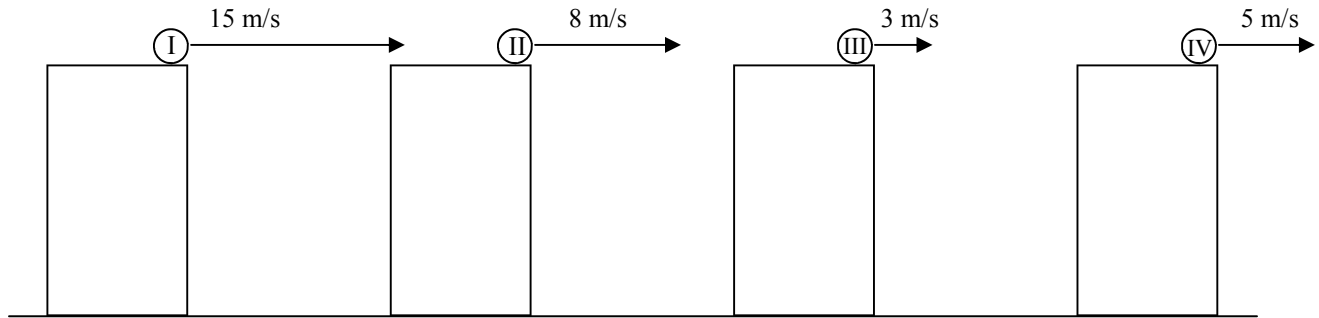


$$\sin \theta = \frac{7}{10}$$

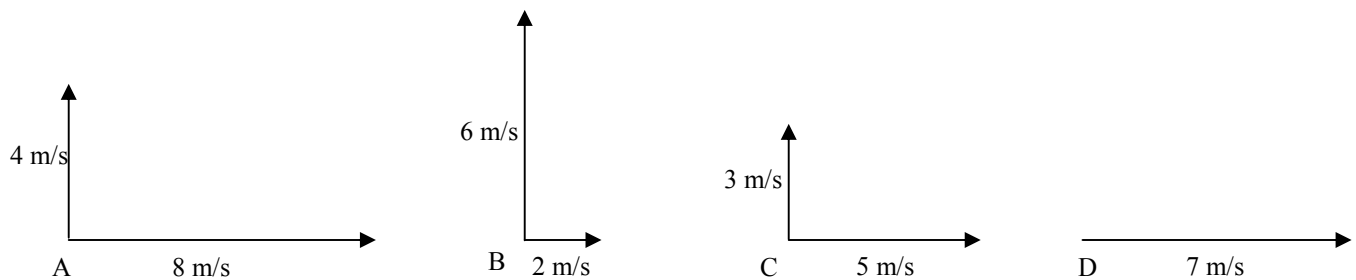
$$\theta = 44.4^\circ \text{ E of N OR } 45.6^\circ \text{ from } +x \text{ axis (} 0^\circ \text{)}$$

this is from y-axis ↑ From X-axis

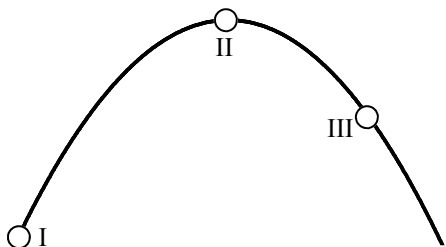
Projectile Concepts



1. Four objects are shot horizontally off of tables of the same height. The arrow lengths show velocity magnitude.
  - A. Rank them from greatest time to least time in the air.
  - B. Rank them from greatest to least range (how far away they land).



2. Four objects are shot at different speeds and angles. The x and y components are given.
  - A. Rank them from most time in the air to least time in the air.
  - B. Rank them from greatest initial velocity to least.
  - C. Which has the greatest range: A or B?



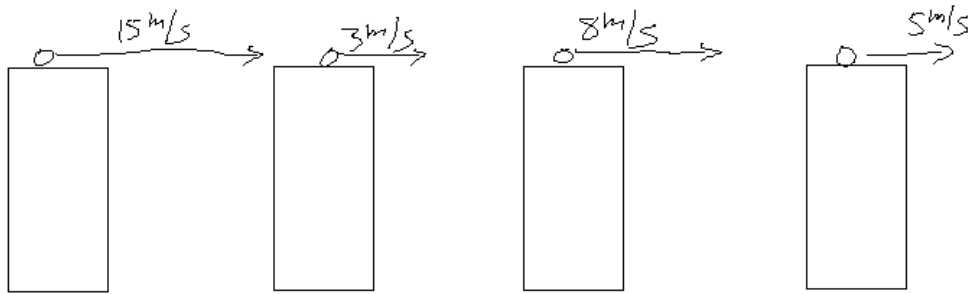
3. The diagram shows a projectile shot from ground to ground. For the following questions answer positions A, B, C, the same, or some combination.
  - A. \_\_\_\_\_ Where the speed (total speed) is greatest.
  - B. \_\_\_\_\_ Where the vertical acceleration is least.
  - C. \_\_\_\_\_ Where the net force is greatest.
  - D. \_\_\_\_\_ Where the vertical speed is greatest.
  - E. \_\_\_\_\_ Where the horizontal speed is smallest.
  - F. \_\_\_\_\_ Where the horizontal acceleration is zero.
  - G. \_\_\_\_\_ Where the direction is zero degrees.
  - H. \_\_\_\_\_ Where the total speed equals the vector addition of the vertical and horizontal speeds.

Four objects are shot horizontally off of equal height tables.

A. rank them from greatest time in the air to least time in the air.

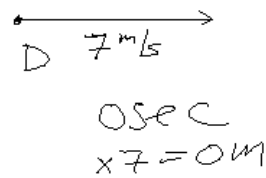
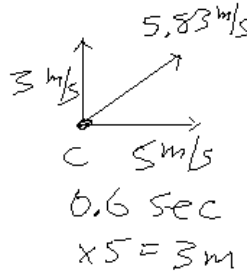
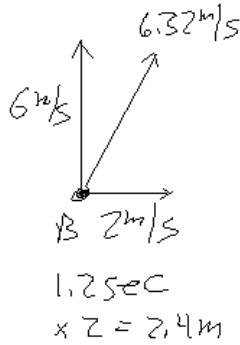
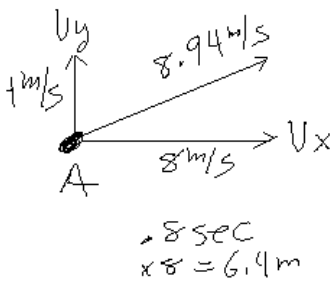
Same for all

they fall from the same heights, so have same time to ground.



B. Rank them from greatest range to least range.

Since they all have the same height and time, the one with the fastest horizontal speed goes farther. SO: 15, 8, 5, 3.



Four objects are shot. The x and y components of their velocities are shown.

A. Rank them from most time in the air to the least time in the air.

One with the greatest y component is in the air the longest. SO: B, A, C, D

D has no y-component so is shot straight into the ground.

B. Rank them from greatest initial velocity to least. The actual initial velocity is the vector addition

(pythagorean theorem) of the two components. See diagrams.

C. Rank them from greatest to least range. : A, C, B, D

Since you have the x velocity of each, all you need is the time in the air for each. Ex. for A. (round g, for speed)

$$v_f = v_i + at$$

$$-4 = 4 - 10t$$

$$-8 = -10t$$

$$t = .8 \text{ sec}$$

$$\text{then } S = \frac{D}{T}$$

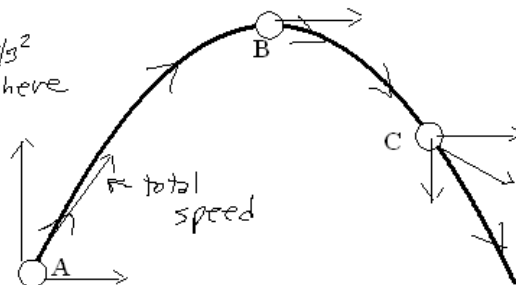
in x-dir, so

$$D = ST$$

$$= 8(.8) = 6.4 \text{ m}$$

The picture shows three positions on the path of a projectile. Answer which position is correct for the following statements. There can be more than one answer or no answers.

- A. A Where the speed (total speed) is greatest.
- B. same Where the vertical acceleration is least.  $g = 9.8 \text{ m/s}^2$
- C. same Where the net force is greatest. =  $F_w$  every where
- D. A Where the vertical speed is greatest.
- E. same Where the horizontal speed is smallest.  $v_x$  is const.
- F. same Where the horizontal acceleration is zero.
- G. B Where the direction is zero degrees.
- H. same Where the total speed equals the vector addition of the vertical and horizontal speeds. See prev. section



### Projectile Motion Problems:

- Shot ground to ground at an angle of  $34^\circ$  above the horizontal and a velocity of 56m/s.
  - Find total range.
  - How high did it go?
- Shot ground to ground with 712 m/s at  $60^\circ$ .
  - How high does it go?
  - How far away does it land?
- A bicyclist is riding 12 m/s across a flat roof 8.5m above the ground.
  - How much hang time did he have when he leaves the building?
  - How far away does he land?
- A football is kicked with a velocity of 14 m/s and an angle of  $48^\circ$ .
  - How high does it go?
  - How far down field does it land?
- Shot horizontally from 125cm up. Lands 1.5 m away. How fast does it leave the projectile launcher?
- Dropped from a plane going 120 m/s at an altitude of 280m. The plane is moving horizontally.
  - How much time for it to hit the ground.
  - How far away will it land from where it was dropped?
- I'm laying on the ground and my lunch decides to be resurrected. I blow chunks at an angle of  $75^\circ$  and 3.4m/s.
  - How high up do they go?
  - How far away do they land?
  - How nasty do they smell?
  - Why are you still reading this?
- Bubba Joe has a really bad cold, but no hankie. So, in true down-home style, he turns his head up and blows his nose. Turns out he blows exactly horizontally (and you said he had no talent). He hits a poor, undeserving toad that is just sitting on a rock, 2.6m away. If Bubba Joes nostril is exactly 1.86 m from the ground, how fast did the green globs of goo leave his breathing passages?
- Disgusted with his last attempt at a witty, but still academically useful projectile motion question, Mr. Murray crumples up his paper and throws it 3.1 m/s and  $35^\circ$  toward the recycle bin (even tired, he is still environmentally conscious). Pretending he is throwing from the floor and to the floor, how high up does it go and how far away does it land?
- It is Friday evening and I am very tired right now. So I am only going to launch one more thing: a final thought. This is exacting work. There is no room for error. Do not assume that by getting one problem right, you will get every one. Work hard. Repeat the problems until you can do them easily and with no errors. - Mr. Murray

### Projectile Motion Problems:

- Shot ground to ground at an angle of  $34^\circ$  above the horizontal and a velocity of 56m/s.
  - Find total range.  $V_x = 46.43$ ;  $V_y = 31.315$  m/s  $t = 6.391$  sec  $\Delta x = 296.71$ m
  - How high did it go?  $\Delta y = 50.03$  m
- Shot ground to ground with 712 m/s at  $60^\circ$ .
  - How high does it go?  $V_y = 616.61$ m/s  $V_x = 356$ m/s  $\Delta y = 19398.36$ m
  - How far away does it land?  $t = 125.84$ sec  $\Delta x = 44798.7$ m
- A bicyclist is riding 12 m/s across a flat roof 8.5m above the ground.
  - How much hang time did he have when he leaves the building?  $t = 1.317$ sec
  - How far away does he land?  $\Delta x = 15.80$ m
- A football is kicked with a velocity of 14 m/s and an angle of  $48^\circ$ .
  - How high does it go?  $V_y = 10.4$ m/s  $V_x = 9.368$ m/s  $\Delta y = 5.523$ m
  - How far down field does it land?  $t = 2.123$ sec  $\Delta x = 19.89$ m
- Shot horizontally from 125cm up. Lands 1.5 m away. How fast does it leave the projectile launcher?  
Convert to meters so  $\Delta x = 1.5$ m and  $\Delta y = -1.25$ m.  $t = 0.505$ sec (y-dir) so  $V_x = V = 2.97$ m/s
- Dropped from a plane going 120 m/s at an altitude of 280m. The plane is moving horizontally.
  - How much time for it to hit the ground.  $V_y = 0$  m/s (horiz.)  $V_x = 120$  m/s  $t = 7.559$
  - How far away will it land from where it was dropped?  $\Delta x = 907.08$ m
- I'm laying on the ground and my lunch decides to be resurrected. I blow chunks at an angle of  $75^\circ$  and 3.4m/s.
  - How high up do they go?  $V_y = 3.284$ m/s  $V_x = 0.88$ m/s  $\Delta y = 0.55$ m
  - How far away do they land?  $t = 0.67$  sec  $\Delta x = 0.59$  m
  - How nasty do they smell? Like a fine bouquet of rot. Mmmmm.
  - Why are you still reading this? Because you are a sicko, too.
- Bubba Joe has a really bad cold, but no hankie. So, in true down-home style, he turns his head up and blows his nose. Turns out he blows exactly horizontally (and you said he had no talent). He hits a poor, undeserving toad that is just sitting on a rock, 2.6m away. If Bubba Joes nostril is exactly 1.86 m from the ground, how fast did the green globs of goo leave his breathing passages?  
 $V_y = 0$ m/s.  $\Delta y = -1.86$  m From y-direction  $t = 0.616$ sec  $V_x = V = 4.22$  m/s
- Disgusted with his last attempt at a witty, but still academically useful projectile motion question, Mr. Murray crumples up his paper and throws it 3.1 m/s and  $35^\circ$  toward the recycle bin (even tired, he is still environmentally conscious). Pretending he is throwing from the floor and to the floor, how high up does it go and how far away does it land?  
 $V_y = 1.778$  m/s  $V_x = 2.539$ m/s;  $t = 0.363$  sec  $\Delta x = 0.92$  m  
How high:  $V_{yi} = 1.778$  m/s  $V_{yf} = 0$  m/s (at the top);  $\Delta y = 0.16$  m
- It is Friday evening and I am very tired right now. So I am only going to launch one more thing: a final thought. This is exacting work. There is no room for error. Do not assume that by getting one problem right, you will get every one. Work hard. Repeat the problems until you can do them easily and with no errors. - Mr. Murray