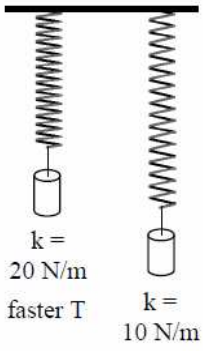
Name:_____

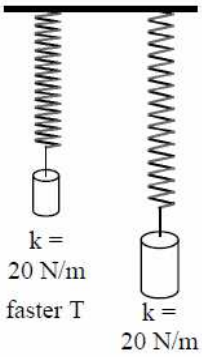
20

# Spring-Mass Systems

**Spring Constant (k in N/m)** — The spring constant tells you how strong (stiff) a spring is. A stiffer spring has a bigger k.




*With the same mass the stronger spring (bigger k) will vibrate faster (smaller T).*




*With the same spring constant more mass causes a slower vibration (larger T).*

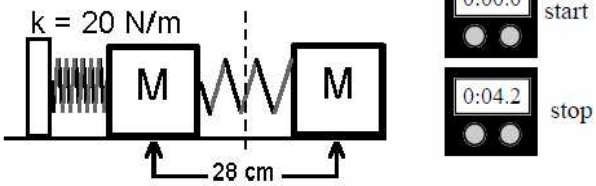
**Harmonic Motion Basics** -

**Amplitude** (A) - maximum displacement from the equilibrium position. The amount of energy in a spring-mass system is determined ONLY by the amplitude.

**Period** (T) - time for one complete cycle.

**Frequency** (f) - number of cycles in one second.

*Amplitude = 14 cm*
*Period (T) = 4.2 sec*




## Hooke's Law:

*Force (in N) of the spring* → $F = -kx$

*Spring Constant (in N/m): bigger k = stiffer spring.*

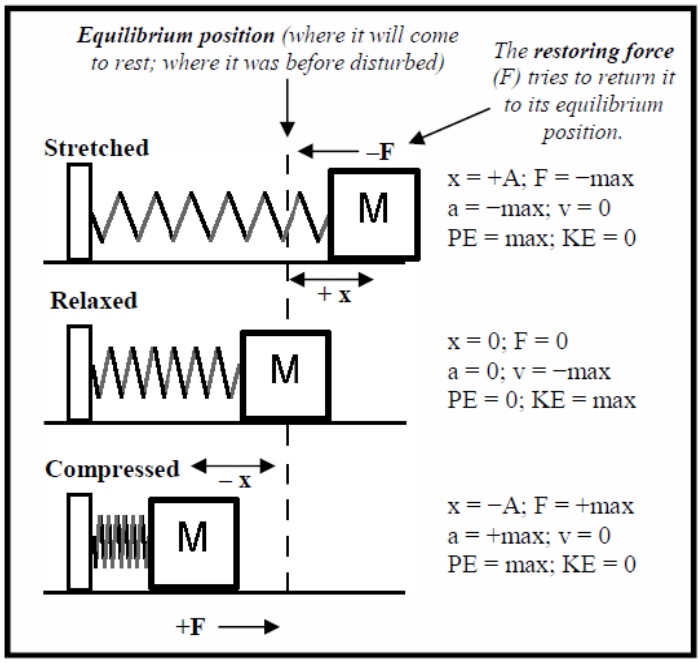
*displacement (in m) from the equilibrium position.*

**Force and Position:** As seen above in Hooke's Law F and x always oppose each other. If F is positive, x is negative, etc. [k, though, is always positive]).

**Stretched Spring**—The spring gets bigger, so x is positive and F is negative.
**Compressed Spring**— The spring gets smaller, so x is negative and F is positive.
**At equilibrium**: Regardless if it is moving or not, at the equilibrium position the spring does not apply a force. x = 0 and F = 0.

**Acceleration (a), Velocity (v), and Energy (E)**
It should be obvious that where the force is zero (equilibrium position), the acceleration is zero. At the ends the mass stops for an instant, so v = 0 and KE = 0, but PE is a maximum ($PE_{el} = \frac{1}{2}kx^2$). The mass speeds up as it moves toward the center, where it has maximum v and KE and minimum PE.

A spring-mass system is called **Simple Harmonic Motion (SHM)** because the force is directly proportional to the displacement of the spring. Anything that follows this rule is known as Simple Harmonic Motion. (A pendulum is only close to simple harmonic motion.)




## Period of a Spring-Mass System:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

*Period (in sec)* — *Mass (in kg)* — *Spring constant (in N/m)*

Notice that amplitude is not in the equation. Just as in all harmonic motion—amplitude does not affect the period or frequency of a spring-mass system.

*Ex. A 350 g mass is attached to a spring that has a spring constant of 12 N/m. What is the period of vibration?*

Variables:
m = 0.35 kg
(1000 g = 1 kg)
k = 12 N/m
T = ____

$T = 2\pi\sqrt{\frac{m}{k}}$

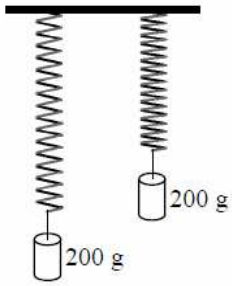
$T = 6.28\sqrt{\frac{.35}{12}}$

$T = 6.28\sqrt{.0292}$

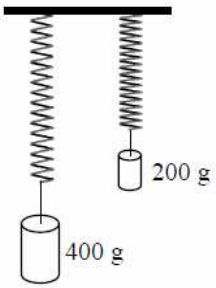
$T = 6.28(.1709)$

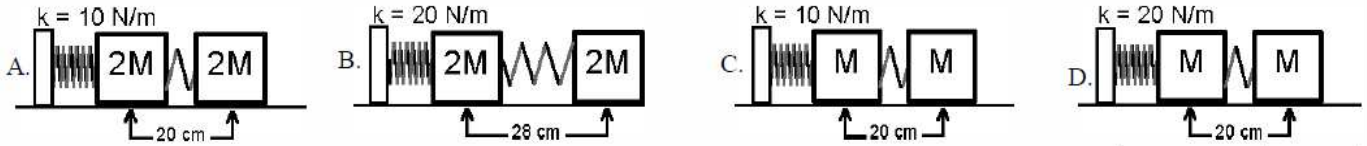
$T = 1.07\,\text{sec}$

1. Which of the springs has the bigger spring constant?
2. How do you know for certain?
3. Which will vibrate faster?

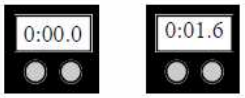



4. Two different masses are suspended from springs with the same spring constant, which will have the faster period?
5. Why?



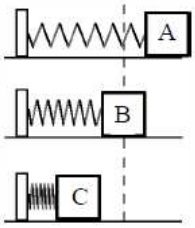




6. What is the amplitude of spring A?
7. What is the amplitude of spring B?
8. A or B will have the fastest period?
9. A or C will have the slowest period?
10. What is the period of spring D?



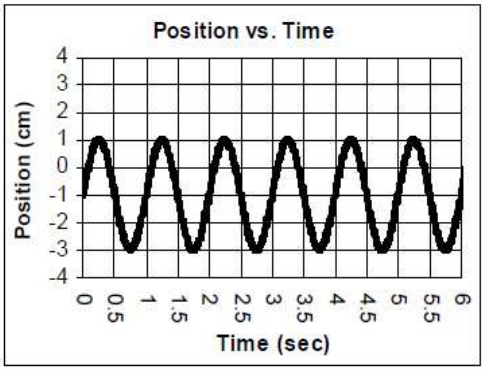

11. Find the mass on spring D.

12. Which position or positions?
    A. ____ Has maximum velocity?
    B. ____ Maximum + force?
    C. ____ Acceleration = 0?
    D. ____ Maximum kinetic energy?
    E. ____ No potential elastic energy?
    F. ____ X is a negative maximum?
    G. ____ Zero amplitude.




13. Positive or negative displacement (x):
    A. ____ A compressed spring (the spring gets shorter)
    B. ____ A stretched spring (the spring is longer)
    C. ____ Pulling on a spring so it stretches.
    D. ____ Pushing on a spring so that it compresses.
    E. ____ Hanging a mass on a spring.

14. In $F = -kx$, is F the spring or what's pulling on the spring?
15. If I pull on a spring with 20 N, then the spring pulls back with how much force?
16. A 5 kg object stretches a spring 20 cm.
    A. How much force is pulling on the spring ?
    B. Find the spring constant.
17. A 300 g mass stretches a spring 50 cm, find the spring constant.
18. Find the period of a spring mass system that has a 5.4 kg mass and a 60 N/m spring constant.




19. What is the amplitude of the above graph?
20. What is the period of the above graph?
21. If it has a 3 kg mass on it, what is its spring constant?